

Fresh inflation with increasing cosmological parameter

Mauricio Bellini*

Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, AP:2-82, (58041) Morelia, Michoacán, México.

I study a fresh inflationary model with an increasing F -cosmological parameter. The model provides sufficiently e-folds to solve the flatness/horizon problem and the density fluctuations agree with experimental values. The temperature increases during fresh inflation and reach its maximum value when inflation ends. This fact would provide a transition between inflation and the radiation-dominated epoch, once inflation ends. I find that entropy perturbations always remain below 10^{-4} during fresh inflation and become negligible when fresh inflation ends. Hence, the adiabatic fluctuations dominate the primordial spectrum at the end of fresh inflation.

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Although a justification from first principles for dissipative effects has not been firmly achieved in the framework of inflation, such effects should not be ruled out on the basis of readiness alone. Much work can be done on phenomenological grounds as, for instance, by applying nonequilibrium thermodynamic techniques to the problem or even studying particular models with dissipation. An interesting example of the latter case is the warm inflationary picture [1,2]. As in new inflation [3,4], a phase transition driving the universe to an inflationary period dominated by the scalar field potential is assumed. However, a standard phenomenological frictionlike term $\Gamma\dot{\phi}$ is inserted into the scalar field equation of motion to represent a continuous energy transfered from ϕ to the radiation field. This persistent thermal contact during inflation is so finely adjusted that the scalar field evolves all the time in a damped regime generating an isothermal expansion. As a consequence, the subsequent reheating mechanism is not needed and thermal fluctuations produce the primordial spectrum of density perturbations [5]. More recently was demonstrated that isentropic and warm pictures are just extreme cases of an infinite two-parametric family of possible inflationary scenarios [6]. Some attempts of a fundamental justification to warm inflation has also been developed [7]. As it appears, its unique negative aspect is closely related to a possible thermodynamic fine-tuning, because an isothermal evolution of the radiative component is assumed from the very beginning in some versions of warm inflation. In other words, the thermal coupling acting during inflation is so powerful and finely adjusted that the scalar field decays ensuring a constant temperature even considering the exponential expansion of the universe.

Very recently an new scenario called fresh inflation was introduced [8]. It can be viewed as a unification of both chaotic [9] and warm inflation scenarios. As in chaotic inflation the universe begins from an unstable primordial matter field perturbation with energy density nearly M_p^4 ($M_p = 1.2 \times 10^{19} \text{ GeV}$ is the Planckian mass) and chaotic initial conditions. Furthermore, initially the universe there is no thermalized so that the radiation energy density when inflation starts is zero [$\rho_r(t = t_0) = 0$]. As initial time we understand the Planckian time $G^{1/2}$, where G is the gravitational constant. Later, the universe will describe a second-order phase transition. Particle production and thermalization occur together during the rapid expansion of the universe, so that the radiation energy density grows during fresh inflation ($\dot{\rho}_r > 0$). The interaction between the inflation field and the particles produced during inflation provides slow-rolling of the inflaton field. So, in the fresh inflationary model (also in warm inflation), the slow-roll conditions are physically well justified. The decay width of the ϕ -field grows with time, so when the inflaton approaches to the minimum of the potential there is no oscillation around the minimum energetic configuration. Hence, the reheating period does not happen in fresh inflation. This model attempts to build a bridge between the standard and warm inflationary models, beginning from chaotic initial conditions which provide naturality. We describe fresh inflation with a Lagrangian for a ϕ -scalar field minimally coupled to gravity, which also interacts with another ψ -scalar field by means of $\mathcal{L}_{int} = -g^2\phi^2\psi^2$, is

$$\mathcal{L} = \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} (\nabla\phi)^2 - V(\phi) + \mathcal{L}_{int} \right], \quad (1)$$

where $R = 6(a\ddot{a} + \dot{a}^2)/a^2$ is the scalar curvature, a is the scale factor of the universe and g is the determinant of the metric tensor $g^{\mu\nu}$ with $\mu, \nu = 0, 1, 2, 3$. In this paper I consider a Friedmann-Robertson-Walker (FRW) metric for a

*E-mail address: mbellini@mdp.edu.ar

spatially flat, isotropic and homogeneous universe described by the line element $ds^2 = dt^2 - a^2 dr^2$. If $\delta = \dot{\rho}_r + 4H\rho_r$ describes the interaction between the inflaton and the bath for a $\gamma = 4/3$ -fluid which expands with a Hubble parameter $H = \dot{a}/a$ and radiation energy density ρ_r , hence the equations of motion for ϕ and radiation energy density are

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \frac{\delta}{\dot{\phi}} = 0, \quad (2)$$

$$\dot{\rho}_r + 4H\rho_r - \delta = 0. \quad (3)$$

Here, $\delta = \Gamma(\theta)\dot{\phi}^2$ describes a Yukawa interaction and the ϕ -decay width is $\Gamma(\theta) = [g_{\text{eff}}^4/(192\pi)]\theta$ [10]. Furthermore, $\theta \sim \rho_r^{1/4}$ is the temperature of the bath. The cosmological parameter $F = (p_t + \rho_t)/\rho_t$ describes the evolution of the universe during inflation

$$F = -\frac{2\dot{H}}{3H^2} = \frac{\dot{\phi}^2 + \frac{4}{3}\rho_r}{\rho_r + \frac{\dot{\phi}^2}{2} + V}, \quad (4)$$

where the total pressure and energy density are given respectively by $p_t = \dot{\phi}^2/2 + \rho_r/3 - V(\phi)$ and $\rho_t = \rho_r + \dot{\phi}^2/2 + V(\phi)$. In previous works [8] only was considered the case where the cosmological parameter F is a constant. However, as we can see in eq. (4), during inflation the potential energy density decreases, so that the radiation energy density becomes more important in F . This means that F must be increasing during fresh inflation, but of course, always remaining below $4/3$, which corresponds to a radiation dominated universe. We can write ρ_r and $V(\phi)$ as a function of ϕ [8]

$$\rho_r = \left(\frac{3F}{4-3F}\right)V - \frac{27}{8}\left(\frac{H^2}{H'}\right)^2 \frac{F^2(2-F)}{(4-3F)}, \quad (5)$$

$$V(\phi) = \frac{3}{8\pi G} \left[\left(\frac{4-3F}{4}\right)H^2 + \frac{3\pi G}{2}F^2\left(\frac{H^2}{H'}\right)^2 \right], \quad (6)$$

where F is a function of ϕ and the time evolution of ϕ is described by the equation

$$\dot{\phi} = -\frac{3H^2}{2H'}F(\phi). \quad (7)$$

We consider in (6) the potential $V(\phi) = [\mathcal{M}^2(0)/2]\phi^2 + [\lambda^2/4]\phi^4$, where $G = M_p^{-2}$ is the gravitational constant and $M_p = 1.2 \cdot 10^{19}$ GeV is the Planckian mass. The inflaton field is really an effective field described by $\phi = (\phi_i \phi_i)^{1/2}$. Furthermore, $\mathcal{M}^2(0)$ is given by \mathcal{M}_0^2 plus renormalization counterterms in the initial potential $\frac{1}{2}\mathcal{M}_0^2(\phi_i \phi_i) + \frac{\lambda^2}{4}(\phi_i \phi_i)^2$ [11], the effective potential is $V_{\text{eff}}(\phi) = [\mathcal{M}^2(\theta)/2]\phi^2 + [\lambda^2/4]\phi^4$. Here, θ is the temperature and $\mathcal{M}^2(\theta) = \mathcal{M}^2(0) + \frac{(n+2)}{12}\lambda^2\theta^2$, such that $V_{\text{eff}}(\phi, \theta) = V(\phi) + \rho_r(\theta, \phi)$. The temperature increases with the expansion of the universe because the inflaton transfers radiation energy density to the bath with a rate larger than the expansion of the universe. So, the number of created particles n [for $\rho_r = (\pi^2/30)g_{\text{eff}}\theta^4$], is given by

$$(n+2) = \frac{2\pi^2}{5\lambda^2 g_{\text{eff}}} \frac{\theta^2}{\phi^2}, \quad (8)$$

where g_{eff} denotes the effective degrees of freedom of the particles and it is assumed that ϕ has no self-interaction.

On the other hand, the scalar metric perturbations are related with density perturbations of the inflaton field. These are spin-zero projections of the graviton, which only exist in nonvacuum cosmologies. The issue of gauge invariance becomes critical when we attempt to analyze how the scalar metric perturbations produced in the very early universe influence a spatially flat isotropic and homogeneous background FRW metric in a coordinate-independent manner at every moment in time. The results do not depend on the gauge when the metric is represented by $ds^2 = (1+2\psi)dt^2 - a^2(1-2\Phi)\delta_{ij}dx^i dx^j$. I will consider the case where the tensor T_{ij} is diagonal, i.e., for $\psi = \Phi$ [12]. A stochastic approach to Φ in the framework of standard inflation was studied in [13]. Furthermore, gauge invariant metric fluctuations has been subject of study also in the framework of warm inflation [14]. The equation that describes the evolution for the modes Φ_k (with wavenumber k), of the field Φ , is [14]

$$\ddot{\Phi}_k + (4+3c_s^2)H\dot{\Phi}_k + \left[2\dot{H} + 2H^2(1+c_s^2)\right]\Phi_k + \frac{c_s^2 k^2}{a^2}\Phi_k = \frac{4\pi}{M_p^2}\tau\delta S, \quad (9)$$

where $c_s^2 = \dot{p}_t / \dot{\rho}_t$. The right-hand side of this equation accounts for the entropy perturbations δS . The evolution of the curvature perturbation \mathcal{R} , is related to the source $\tau\delta S$ [14]

$$\begin{aligned} \tau\delta S = & -\frac{M_p^2}{4\pi} \left(H\Phi + \dot{\Phi} \right) A - \frac{M_p^2 (1 - c_s^2)}{4\pi a^2} k^2 \Phi \\ & - \frac{2}{3} \rho_r \left(\frac{4aV'v}{k\dot{\phi}} + \frac{\delta\rho_r}{\rho_r} \right), \end{aligned} \quad (10)$$

where v originates from the decomposition of the velocity field as $\delta U_i = -(iak_i/k)v e^{i\vec{k}\cdot\vec{x}}$ (see Bardeen [15]) and $A = \frac{8\rho_r(H+V'/\dot{\phi})-2\Gamma\dot{\phi}^2}{3(\dot{\phi}^2+4/3\rho_r)}$. For strong dissipation (i.e., for $\Gamma \gg H$), $A \simeq -2\Gamma$ and the relevant term in $\tau\delta S$ is

$$\tau\delta S \simeq \frac{M_p^2}{2\pi} \left(H\Phi + \dot{\Phi} \right) \Gamma. \quad (11)$$

Hence, the long wavelength metric fluctuations in the strong dissipative limit are well described by [see eqs. (9) and (11)]

$$\begin{aligned} & \ddot{\Phi}_k + \left[(4 + 3c_s^2) H - 2\Gamma \right] \dot{\Phi}_k \\ & + \left[\frac{c_s^2 k^2}{a^2} + 2 \left[\dot{H} + H^2(1 + c_s^2) - H\Gamma \right] \right] \Phi_k = 0. \end{aligned} \quad (12)$$

If we decompose the modes Φ_k as $\Phi_k = a^{-(2+3c_s^2/2)} \chi_k e^{\int g_k dt}$, the eq. (12) becomes in the following system

$$\ddot{\chi}_k + \left[\frac{c_s^2 k^2}{a^2} - H^2 \left(2 + \frac{9}{4} c_s^4 + c_s^2 \right) - \frac{3c_s^2}{2} \dot{H} \right] \chi_k = 0, \quad (13)$$

$$2g_k \dot{\chi}_k + \chi_k (g_k^2 + \dot{g}_k) = \left[3(c_s^2 + 2) \Gamma H + \Gamma^2 - \dot{\Gamma} \right] \chi_k. \quad (14)$$

As we can see in eq. (14), the function $g_k(t)$ only takes into account the thermal effects, which are product of the interaction of the inflaton field. Furthermore, this function is coupled with χ_k , such that $a^{-(2+3c_s^2/2)} \chi_k(t)$ only takes into account the adiabatic fluctuations of the metric Φ . In the classical limit for the modes we obtain $\left| \frac{\dot{\chi}_k}{\chi_k} \right| \ll 1$ [16,17], so that we can approximate (14) to $g^2 + \dot{g} = (c_s^2 + 2) \Gamma H + \Gamma^2 - \dot{\Gamma}$. Note that now g becomes independent of the wavenumber k . To long wavelength perturbations, $\dot{\mathcal{R}}$ is mainly described by its dissipative term [14]

$$\dot{\mathcal{R}} \Big|_k \simeq \frac{4\rho\Gamma}{3(\rho_t + p_t)} \Phi_k. \quad (15)$$

When the horizon entry, i.e., for $k = aH$, one obtains [14]: $\dot{\mathcal{R}} \Big|_{k=aH} = \frac{\rho_t \Gamma}{3(\rho_t + p_t)} \Phi_{k=aH}$. This expression can be written as a function of the cosmological parameter $F = \frac{p_t + \rho_t}{\rho_t}$ [see eq. (4)], or as a function of H and its time derivative: $\dot{\mathcal{R}} \Big|_{k=aH} = -\frac{2H^2\Gamma}{H} \Phi_{k=aH}$. It is useful to describe the spectrum in terms of the spectral index n_s . It was obtained for warm inflation in [14] in terms of the slow-roll parameters

$$\begin{aligned} n_s = & 1 - \frac{1}{2(1 - \epsilon + \alpha)} \left[\frac{(11 + 5\alpha)\epsilon}{2(1 + \alpha)} - 3\eta \right] \\ & - \frac{M_p^2(1 + 7\alpha)\Gamma'H'}{48\pi\alpha(1 + \alpha)(1 - \epsilon + \alpha)H^2}, \end{aligned} \quad (16)$$

where $\alpha = \frac{\Gamma}{3H}$, and $\epsilon = \frac{M_p^2}{4\pi} \left(\frac{H'}{H} \right)^2$, $\eta = \frac{M_p^2}{4\pi} \left(\frac{H''}{H} \right)$ are the slow-roll parameters in standard inflation [18].

As an example we consider the particular case where the Hubble and cosmological parameters evolve respectively as $H(\phi) = A \phi^2$ and $F(\phi) = B \phi^{-2}$, where A and B are respectively $G^{1/2}$ and G^{-1} dimensional constants given by $A^2 = \frac{\lambda^2}{4} \frac{8\pi G}{3}$ and $B = \frac{3\lambda + \sqrt{9\lambda^2 + 48\pi G M^2(0)}}{3\lambda\pi G}$. From eq. (7) we obtain the temporal evolution for the inflaton field: $\phi(t) = \phi^{(s)} e^{-\frac{3AB}{2}t}$, where $\phi^{(s)}$ is its initial value. It must be sufficiently large to assures at least, the 50 – 60 e-folds needed during inflation before transurred ($10^{10} - 10^{12}$) Planckian times. Since $H = \dot{a}/a$, the time evolution of

the scale factor will be $a = a_0 e^{-\frac{\phi^2(t)}{3B}}$, which increases with time due to the decreasing of $\phi(t)$. Furthermore, the temperature, written as a function of the field, is obtained from the equation (3)

$$\theta(\phi) = \{4\pi\phi(8\phi - 3B) [16\mathcal{M}^2(0) + \phi^2 (8\lambda^2 - 18A^2B) + 9A^2B^2]\} [BAg_{\text{eff}}^4 (4\phi^2 - 3B)]^{-1}, \quad (17)$$

which is zero when fresh inflation starts. The ϕ -value at this time is $\phi^{(s)} = 3B/8$.

Figures (1) and (2) show respectively the evolution of the cosmological parameter $F(t)$ and the temperature $\theta(t)$. These graphics were made using the parameter values $\mathcal{M}(0) = 1.5 \times 10^{-9} \text{ G}^{-1/2}$, $\lambda = 4 \times 10^{-15}$ and $g_{\text{eff}} = 25$. With these parameter values we obtain respectively $A = 0.578 \times 10^{-6} \text{ G}^{1/2}$ and $B = 488726.728 \text{ G}^{-1}$. Notice that fresh inflation ends at $t_e \simeq 2 \times 10^8 \text{ G}^{1/2}$, when $\theta(t_e) \simeq 6 \times 10^{-8} \text{ G}^{-1/2}$ [see Fig. (2)]. The ϕ -value when fresh inflation ends is given by $\phi^{(e)} \equiv \phi(t_e) \simeq 8 \times 10^5 \text{ G}^{-1/2}$. Hence, ϕ takes transplanckian values during fresh inflation with increasing cosmological parameter. Figure (3) shows the evolution of $\dot{\mathcal{R}}|_{k=aH}$. Notice that tends to zero at the end of inflation and always remain below $\dot{\mathcal{R}} \sim 10^{-4}$. Hence, at the end of fresh inflation the entropy perturbations are no longer important and the primordial spectrum of the perturbations is due only to adiabatic (scalar) fluctuations $\langle \delta\phi^2 \rangle \sim 3H\theta/(4\pi)$. This result is in agreement with de Oliveira and Jorás [14]. Fig. (4) and (5) show respectively the function $g_k(t)$ (when it is decoupled with χ_k) and the adiabatic mode of the fluctuations $a^{-(2+3c_s^2/3)}\chi_{k=0.2}(t)$. Note that g_k decreases at the end of fresh inflation and the mode $a^{-(2+3c_s^2/3)}\chi_{k=0.2}(t)$ oscillates. However, can be showed that for long wavelengths (with respect to the horizon) the modes increase monotonically.

Fig. 1) Temporal evolution of the cosmological parameter F which increases with time, but remains below $4/3$.

Fig. 2) Evolution of the temperature $\theta(t)$ during inflation. The maximum, with parameters $\mathcal{M}(0) = 1.5 \times 10^{-9} \text{ G}^{-1/2}$, $\lambda = 4 \times 10^{-15}$ and $g_{\text{eff}} = 25$, occurs at the end of fresh inflation (i.e., around $t_e \simeq 2 \times 10^8 \text{ G}^{1/2}$).

Fig. 3) Evolution of $\dot{\mathcal{R}}|_{k=aH}$ shows that it goes to zero at the end of fresh inflation.

Fig. 4) Temporal evolution of g_k (when it is decoupled with χ_k).

Fig. 5) Temporal evolution of $a^{-(2+3c_s^2/3)}\chi_{k=0.2}$.

Finally, a calculation shows that the number of created particles reach its asymptotic value $n \simeq 9 \times 10^6$, at the end of the inflationary period. Furthermore, with the parameters here used I find a spectral index n_s very closed to one: $n_s \simeq 0.99999999942$. This value is in very good agreement with recent (BOOMERANG-98, MAXIMA-1 and COBE DMR) observations [19], which are very consistent with a spectral index $n_s \simeq 1$.

To summarize, the model here studied shows that fresh inflation can be a feasible alternative to standard inflation [20,21]. It is true that warm inflation can be problematic from the point of view of its initial thermal conditions, because requires a nonzero thermal component at the beginning of inflation. Fresh inflation attempts to build a bridge between the standard and warm inflationary models, beginning from chaotic initial conditions which provides naturality. An important characteristic of this model is that shows a natural transition between the end of inflation and the epoch when the universe is radiation dominated. The main result here obtained is that entropy perturbations becomes negligible at the end of fresh inflation, so that the adiabatic scalar perturbations dominate the power spectrum at the end of fresh inflation.

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